Mathematical Contest in Modeling and Interdisciplinary Contest in Modeling
Summaries, Pictures and Results
Simpson College
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Lwanda, Maya and Shikha find ways to minimize the wait time at toll booths.

Soooooothe the Booth-Queue

by

Shikha Basnet, Lwanda Manxodidi and Maya Hristakeva

Honorable Mention

To reduce the annoyance of the motorists in the heavily-traveled toll roads we developed a model that attempts to minimize the traffic congestion by spreading the number of vehicles throughout the plaza. In developing this model, we considered the two possible factors that could potentially cause congest, at entrance (part one) and exit (part two). Since having congestion in the exiting part of the plaza will clog the entire traffic we prioritized minimizing congestion in that area.

The first part of the model uses recurrence relations to calculate the waiting times of the vehicles. We also computed the number of vehicles needed to be processed per unit of time. In the second part, we incorporated a curve-fit method by fitting the shape of the exiting half of a toll-booth plaza into an isosceles trapezoid. We then took advantage of the trapezoid's geometric properties and calculus analysis to predict and manage the traffic flow as it exits the toll-booth plaza. Here, we determine the traffic flow from the booths needed to avoid congestion in the exiting half of the plaza for the given period of time.

The results from part one and part two can be used to determine the optimal combination of various types of booths that will regulate traffic so the exiting half of the plaza is not congested. Besides our attempt to minimize congestion we also proposed a suggested merging pattern which seemed to have room for further improvement. According to the model, if a variety of booth types is desired then a larger number of booths than that of highway lanes are needed.

Modeling Flood Damage Caused by a Failure of the Lake Murray Dam

by

Scott Roth, Greg Elliott and Prakash Kayastha

Honorable Mention

Scott and Greg use contour maps to model flood regions.

The Lake Murray Dam in South Carolina does not meet current safety criteria for earthquakes. In the event of an earthquake that breaches the dam, 2.6 billion cubic meters of water will be released. Our project models the flood following a critical failure of the Lake Murray Dam.

We begin by modeling the changes in elevation of the land through analysis of a topographic map. We reconstructed a map from an online source into a large scale paper map in order to closely examine the topography of the land. We divided the map into sections to help us calculate the amount of space susceptible to flooding. We model the flow of water as it emerges from the dam and rushes downstream towards the capitol.

The problem asks that we examine the ramifications of the flood on the South Carolina State Capitol and Rawls Creek. First, we were to determine if the capitol building is in danger in the event of the dam being breached. The findings of the model suggest that water levels from the flood will never reach the capitol but will come within 800 meters of the building. In addition we look specifically at the flooding of Rawls Creek, a tributary of the Saluda River. The model computes the depth and scope of the flood on and around Rawls Creek.

Om, Chris and Nick use Excel to model fresh water supplies.

Two *Super* Models, "Drinking Their Water"
by
Om Gurung, Chris Fink and Nick Phillips
Honorable Mention

Two *super* models, "drinking their water" - what does this mean? Our group was given the basic task of selecting a non-renewable resource and projecting its degradation and depletion over a long horizon. We chose water, and created our two models - "drinking it".

We created the first model to give ourselves a base model to work from. We used this model to project the future depletion of the earth's freshwater supply at a constant rate.

Our second model was adapted from our first model. The model created turns out to be a very simple, elegant and flexible design. We used historical data along with predicted future demographic changes to calculate the projected depletion rate of earth's freshwater supply until the year 2050.

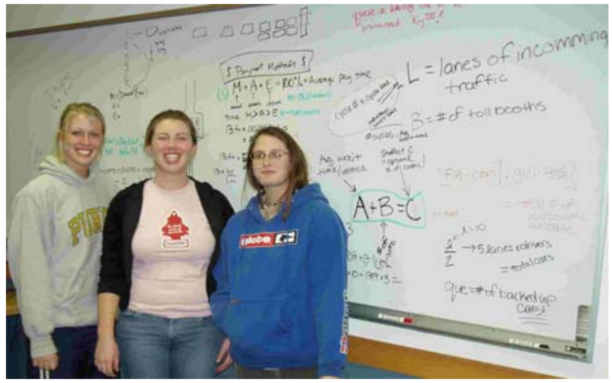
After creating our two models, we addressed various policies that have an effect on freshwater depletion and degradation rates. One of the policies created, included the formation of a global task force. We chose to have this task force uphold various policies relating to freshwater preservation, management, degradation and protection of the global supply.

The modeling paper that follows explains our attempt to accurately predict the global freshwater depletion rate.

Toll Plaza Optimization
by
Kelli Esbaum, Mandi White, Lindsay Saunders
Successful Participant

Mr. Brown left for work this morning at 7:30 AM. He took the 10 lane highway into the city and at 8:00 AM, Mr. Brown was stuck in traffic congestion a mile before the toll plaza. At 9:15 AM, Mr. Brown had made it to the toll plaza and paid his toll at one of the many toll booths. Leaving the toll plaza, Mr. Brown found himself stuck in more traffic congestion as everyone merged back down to 5 travel lanes. Finally, at 10:00 AM, Mr. Brown arrived at work frustrated, irritated, and late. How could this toll plaza have been changed so that Mr. Brown and his fellow motorists traveling during peak traffic hours would have experienced less frustration this morning?

We were given the task of determining the optimal number of toll booths for a given number of highway lanes that would minimize frustration during peak traffic hours. We defined optimal to mean the shortest time waiting in the system with the fewest number of toll booths. Our model divided the time spent in the total system into three parts: time waiting in line before reaching the toll plaza, time spent paying the toll, and time spent merging after the toll plaza. To calculate the first part of the system, we



Kelli, Lindsay and Mandi use probabilities and Excel to create and test models for optimal tollbooth design.

used the M/M/n queue theory and found the mean waiting time in line before the toll plaza. We fit known equations such as Poisson's distribution and the Erlang C function into our model to determine the mean time spent in the queue. To find the time spent paying the toll, we assumed that three different methods of payment could be used. Paying a toll manually would take the most time, electronic pay would take the least time, and automatic pay would be in between. We estimated the percentage of vehicles that would use each method of payment to determine the average time it would take any vehicle to pay one toll. To determine the wait time after the toll, we used vehicle discharge rates and the number of lanes vehicles merged to. We summed the three waiting times to obtain a total mean waiting time in the system. We then added the ratio of toll booths to travel lanes and obtained a score for each scenario. The lowest score determined the optimal number of toll booths for that number of highway lanes.

We found that 8 toll booths for a 10 lane highway were optimal. The optimal number of tollbooths can be found using the equation (tollbooths) = 0.3411(lanes) + 4.1429. This equation only works for up to 26 lanes of highway. 28 or more highway lanes are more optimal when there is exactly one toll booth per travel lane.

Casie, Tracy and Jean optimize wait time in toll booth design.

For Whom the Bar Lifts: Modeling a Toll Plaza

by

Casie Schmitt, Tracy Robson and Jean Clipperton

Successful Participant

Toll plazas are difficult to model because there are multiple variables to account for and others that cannot be accounted for at all. In the construction and/or modification of a toll plaza, it's important to understand both the rate at which customers can be serviced as well as the rate at which they are arriving. After evaluating toll booth operations and the variables involved, we created a model that emphasizes the importance of wait time per customer to determine the most efficient number of booths for the toll plaza. Our model, once wait time, arrival and departure rates are known, can predict the optimal number of booths.

The other issue to address in toll plazas is congestion: abnormally high traffic volume might require more booths, but the additional customers going for the same limited space (the original highway) will leave toll payers sitting in the booths, unable to leave. Thus, the recommended number of booths cannot allow for too great a number of consumers competing for the limited resource (space in a lane). We've implemented a ratio as a means to check our model's results: for any given road the ratio of tollbooths to highway lanes should not exceed 3:1. A gradual return to the original number of lanes, with random release of customers should only leave two to three people vying for the same 'spot' on an exit lane, while the majority of the time it will be closer to one or two. This allows for the greatest number of toll lanes without over-burdening the highway as customers try to exit the plaza.